Sheet No: 3

Isometric Projection

17

17.1. INTRODUCTION

As already discussed, the exact and detailed description of any object can be represented by a number of views obtained on different planes of projection, held mutually at right angles to each other, for *orthographic projections*. Such drawings can be correctly interpreted and quickly visualized only by those persons who have a good knowledge of principles used for these projections. Also their execution requires a definite exercise of constructive imagination.

A detailed study of the theory of orthographic projections will be undertaken in the next chapter.

To make the drawing more understandable, several forms of 'one plane' conventional or projection drawings are used to supplement the orthographic drawings. These one plane drawings, which can be easily understood by persons without any formal technical training, are called *pictorial drawings*. Such drawings reveal faces of an object at once, approximately, as they appear to the observer.

It must be emphasized, however, that so for as engineering is concerned, all one plane drawing methods are normally used as auxiliaries to the standard method of orthographic projections.

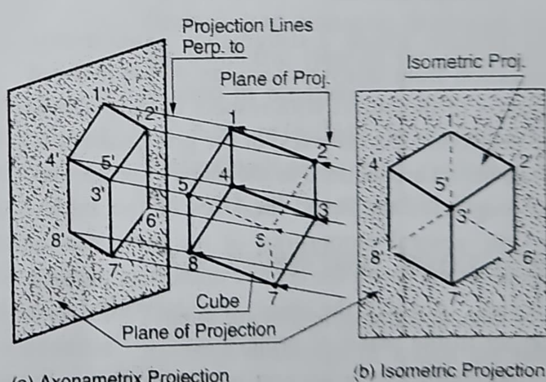
An engineering student is required to be proficient at drawing the *"pictorial views"* of objects and also able to convert given othographic views to pictorial views.

17.2. METHODS OF PROJECTION

Pictorial drawings are projected on a single plane of projection by the methods of:

1. Axonometric Projection.
2. Oblique Projection, and
3. Perspective or Central projection.

Out of these *three types of pictorial drawing* of an object; it can be seen that the perspective is the most realistic but it is tedious to draw, and is not used in machine drawing. The *isometric projection, a particular case of axonometric projection*, though dose not look as pleasings as the perspective, is mostly used because it is easier to draw and dimension, *Fig. 17.1*. The *oblique projection* is used, with advantage, for objects with curved features only on one face or on parallel faces. It is equally easy to draw and dimension as the isometric projection.



(a) Axonametrix Projection

(b) Isometric Projection

Fig. 17.1.

Here, in this chapter we shall deal with the *axonometric projection with special emphasis on the isometric projection only*. The perspective and oblique projections will be dealt in separate chapters.

17.3. AXONOMETRIC PROJECTION

In axonometric projection the object, a cube for example, is placed such that its faces are inclined to the plane of projection, so that its three principal faces may be represented in one view.

When the projectors are perpendicular to the plane of projection, the resulting pictorial projection is a *normal axonometric projection*, otherwise it is an *oblique axonometric projection*. The latter is not used in actual practice, the 'axonometric projection' is generally understood to refer to normal axonometric projection. *The normal axonometric projection*, in a way, *is a branch of orthographic projection, Fig. 17.2.*

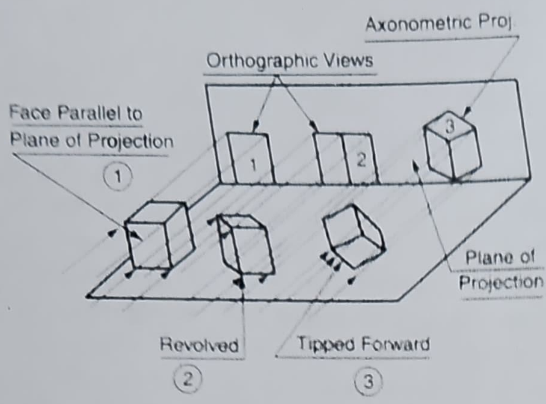


Fig. 17.2. Method of Obtaining Axonometric

17.4. TYPES OF AXONOMETRIC PROJECTION

The feature which distinguishes axonometric projection from other projections, is the inclined position of the object with respect to the plane of the projection. As the axes, or the principal edges, of the object are inclined to the plane of projection, the lengths of the lines, the sizes of the angles, and the general proportions of the projection of the object vary with the infinite number of possible positions in which the object may be placed with respect to the plane of projection.

Axonometric projection is classified as:

1. Isometric projection.
2. Dimetric projection, and
3. Trimetric projection, *Fig. 17.3.*

The theory behind each of these types is same, but the angle of projection for each type is different. *Figure 17.3* shows the three types of axonometric projection. In isometric projection, the three axes form equal angles of 120° to the plane of projection, and only one scale is needed for measurement along each of the three axes. This is the easiest type of axonometric drawing to execute.

In dimetric projection, only two of the three angles are equal, and two special foreshortened scales are required to measure distances along their respective axes, *i.e.*, one scale to do measurements along the axes that foreshorten by same amount, while the second scale for measurements along the third axis, which foreshorten by a different amount, proportionally.

Trimetric projection requires three different foreshortened scales, as all three angles between the three axes are different.

Because of the complexity of constructing dimetric and trimetric drawings, in general, only the isometric type of the axonometric projection is used in practice.

The principle involved in axonometric projection can be best explained by considering the cube, as illustrated in *Fig. 17.2.*

17.5. ISOMETRIC PROJECTION

The isometric projection is used in the preparation of such drawings in which the essential features are required to be shown, otherwise, in three orthographic views. The isometric projection of an object is a single orthographic projection, drawn with the object so oriented with respect to the

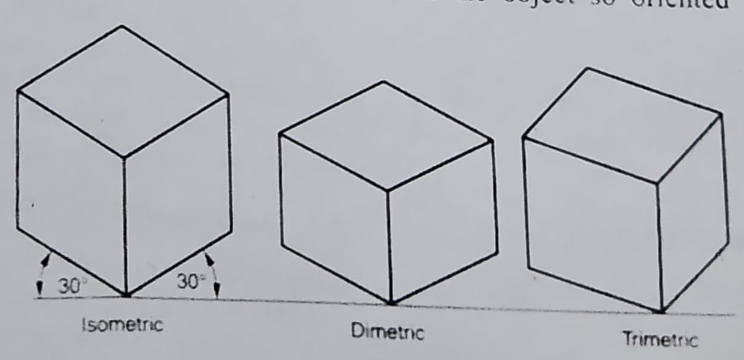


Fig. 17.3. Three types of Axonometric Projections

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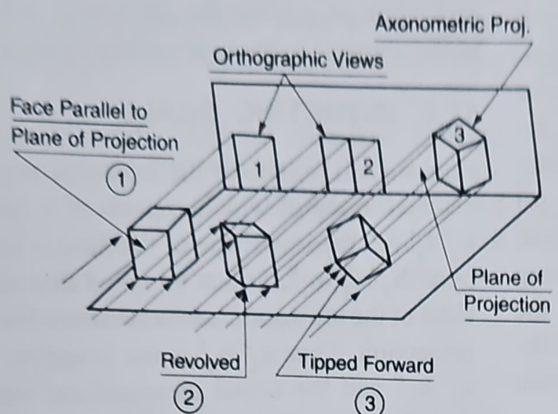

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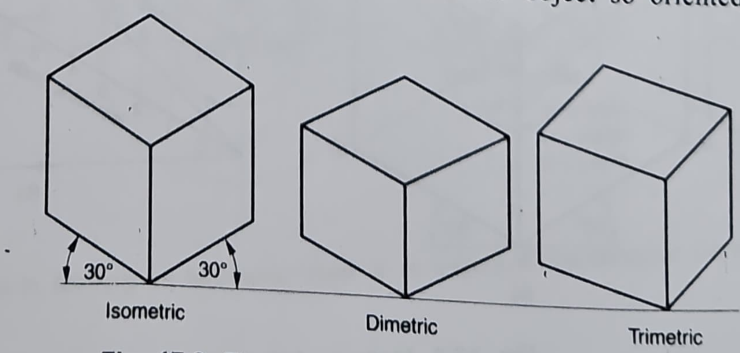


Fig. 17.3. Three types of Axonometric Projections

By looking at the cube from the right side, in *Fig. 17.4. (i) (b)*, and following the first angle projection, it can be seen in the side view that the cube has been rotated to within **35° 16′** of the horizontal centre line.

Geometrically, the principle of isometric projection, by rotation method, is illustrated in *Fig. 17.4. (ii) (a), (b), (c)*, and *(d)*.

Figure 17.4. (ii) (a) shows three orthographic views of a cube, when placed with one of its faces parallel to the principal vertical plane of projection. At (*b*) the three orthographic views of the cube are shown when it has been revolved so that the vertical faces make equal angles (*i.e.*, 45°) with the principal vertical plane of projection. At (*c*) the two views are obtained by rotating the revolved cube in such a way that all the three edges meeting at the front corner 1 are equally inclined to the vertical plane of projection. In this position of the cube the body diagonal 1C will also be perpendicular to the picture plane.

The three front edges are called the *isometric axes* and their angle of inclination with the vertical plane of projection is 35° 16′. The front view obtained in this way shows three principal faces of the cube equal in shape and is called isometric projection. Literally isometric also means *'equal measure'*. In this type of projection the angles between the projections of the front three edges, or between the *three isometric axes*, are 120°, their projected lengths are approximately 82 (actually 81.65%) of their true lengths.

It should be observed that the 90° angles of the cube, in the isometric projection, appear as 120° or 60°. Any line which is parallel to one of the isometric axes is called an *isometric line.* A line, which is not parallel to either of the isometric line, which is not parallel to either of the isometric axes is called *a non-isometric line*. In an isometric protection of a cube the faces of a cube, or any planes parallel to them, are called *isometric planes.*

17.6. ISOMETRIC SCALE

Figure 17.5. (a) shows the isometric projection of a cube the top of which appears as a rhombus **a_1' b_1' f_1' e_1'**, the true shape of this face is represented by **a' b_1'' f_1' e_1''**,. It can be noted that all the four sides of the top square are foreshortened in isometric projection. Thus **a_1' b_1'** is the isometric length of **a_1' b_1''** and the actual measurement would show that the isometric length is 81.65% of the true length, or approximately equal to 82%.

To draw the isometric scale, as shown in *Fig. 17.5. (b)*, from any point on a horizontal line draw two lines inclined at 30° and 45° respectively. Along the 45° line set out a true length scale, and from it draw vertical lines, *i.e.*, 90° to the horizontal line, to cut the 30° line, then the corresponding scale projected on 30° line is the isometric scale.

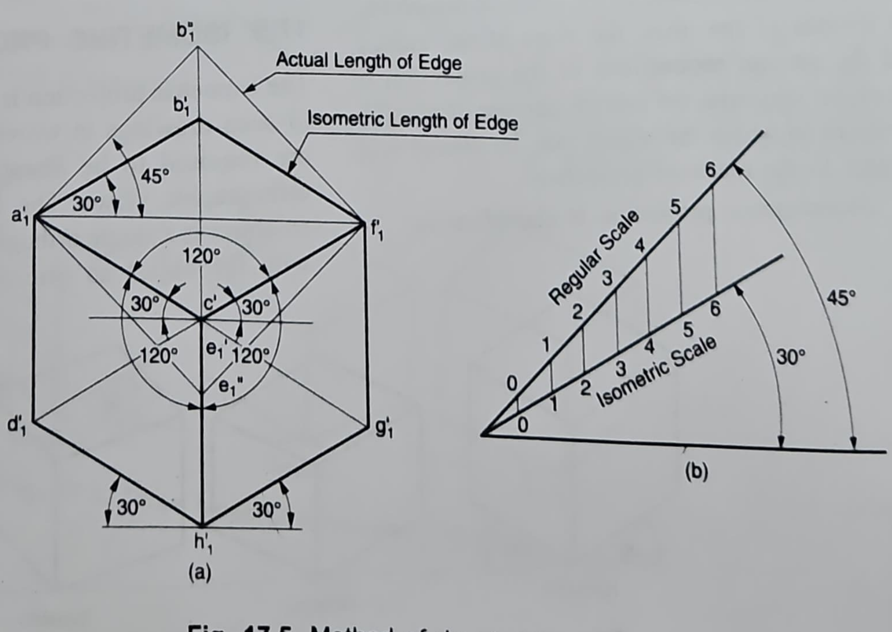
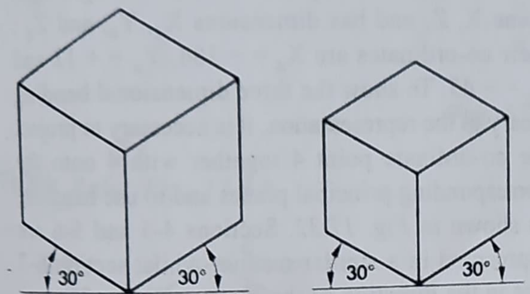


Fig. 17.5. Method of drawing Isometric Scale

17.7. ISOMETRIC DRAWING

As discussed above, the lines in isometric projection do not show the true lengths of the object edges, but are foreshortened by a definite amount. In making such projections, either it should be projected from the orthographic views of the object, as discussed already, or a special isometric scale should be used for transferring measurements. To avoid this tedious construction, if the foreshortening of the lengths is ignored and the true lengths are laid out along the isometric axes or along the isometric lines, the view obtained is called *isometric drawing*.

Figure 17.6 shows that the isometric drawing is slightly larger (22.5%) than the isometric projection. The variation in size does not effect the use of the regular scale, the construction of an isometric drawing is much simpler than the construction of isometric projection.



Isometric Drawing (a)

Isometric Projection (b)

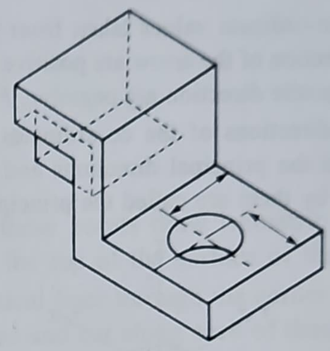
Fig. 17.6. Isometric drawing is 22.5% larger than the Isometric Projection

17.8. RECTANGULAR CONSTRUCTION

The steps in making an isometric drawing of a simple rectangular object are illustrated in *Fig. 17.7*. The axes are first drawn at an angle of 120° with each other, the measurements are made equal size along the axes, and the remaining lines are drawn parallel to their corresponding axes.

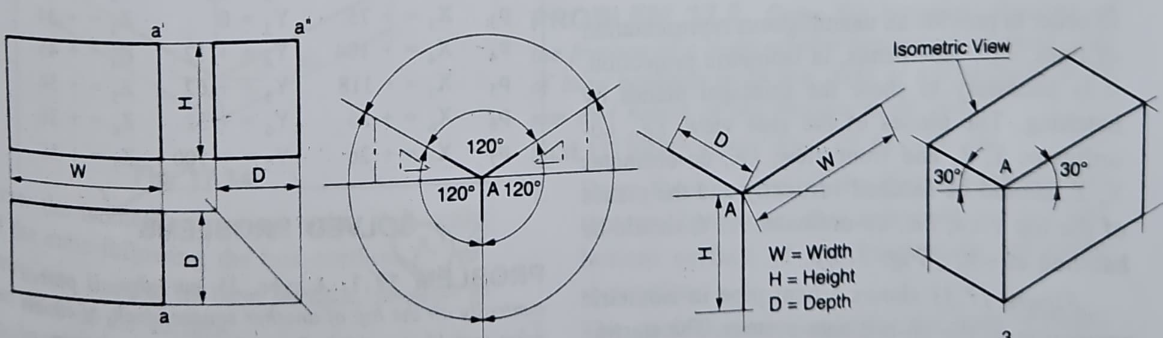
17.9. USE OF INVISIBLE LINES

The use of hidden lines in isometric drawing is governed by the same rule as in all other types of projection, *i.e.*, invisible lines are omitted unless they are needed to make the drawing clear. A case where invisible lines are needed is illustrated in *Fig. 17.8* in which a projecting part can not be clearly described without the use of the invisible or dotted lines.



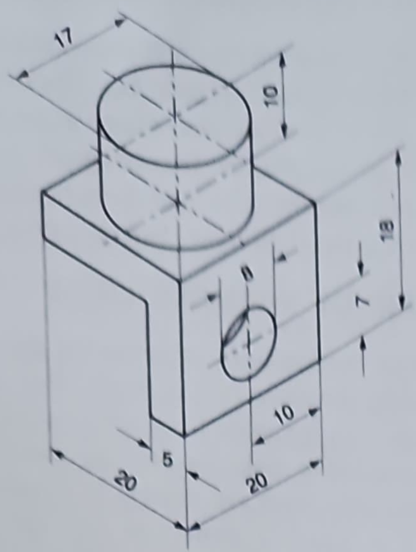
Use of Invisible Lines and Centre Lines

Fig. 17.8.



Given Views 1 2 3

Fig. 17.7. Steps in Making an Isometric View of an object of Rectangular Construction



(b)

Fig. 17.30.

All co-ordinate values taken from the origin in the direction of the arrow are positive and those in the opposite direction are negative, *Fig. 17.31*.

The directions of the co-ordinates X, Y, Z, are called the principal directions and the areas enclosed by them are called the principal planes.

Fig. 17.31.

17.25. REPRESENTATION OF THE SYSTEM OF CO-ORDINATES

In order to provide an unambiguous representation of lines, *e.g.*, pipe bends, in isometric projection, it is necessary to show the principal planes by hatching. The planes of the side view, *i.e.*, co-ordinates Y, Z, and front view, *i.e.*, co-ordinates X, Z, should be hatched vertically and the planes of the top view, *i.e.*, co-ordinates X, Y, should be hatched at –30°, *Fig. 17.32*.

Figure 17.33 shows a bent pipe in isometric projection in the co-ordinate system. The starting point for the drawing and the dimensioning is $1(p_1)$ with co-ordinates $X_1 = 0$, $Y_1 = 0$, $Z_1 = 0$.

The section 1-2 lies on the X co-ordinate and has co-ordinates $X_2 = +50$, $Y_2 = 0$ and $Z_2 = 0$.

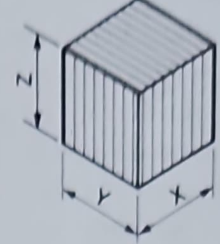


Fig. 17.32.

Section 2-3 lies in the principal plane X, Z and has dimension X_3 and Z_3 and co-ordinates $X_3 = +75$, $Y_3 = 0$ and $Z_3 = +34$. The vertical hatching, in *Fig. 17.32*, shows clearly that the plane of bending of the pipe lines in the principal plane X, Z,

Although, in the representation, section 3-4 is a continuation of 2-3, point 4 is outside the principal plane X, Z, and has dimensions X_4, Y_4, and Z_4; their co-ordinates are $X_4 = +104$, $Y_4 = +12$ and $Z_4 = +45$. To show the three dimensional bending cleary in the representation, it is necessary to project the co-ordinate point 4 together with 4 onto the corresponding principal planes and to use hatching as shown in *Fig. 17.32*. Sections 4-5 and 5-6 are represnted in a similar manner whilst section 6-7 lies in the direction of the Y- co-ordinate. The co-ordinate values for the bent run shown in *Fig. 17.33* are given in Table 17.1.

Table 17.1. Co-ordinate Values For Bent Pipe run

p_1	$X_1 = 0$	$Y_1 = 0$	$Z_1 = 0$
p_2	$X_2 = 50$	$Y_2 = 0$	$Z_2 = 0$
p_3	$X_3 = +75$	$Y_3 = 0$	$Z_3 = +34$
p_4	$X_4 = +104$	$Y_4 = +12$	$Z_4 = +45$
p_5	$X_5 = +118$	$Y_5 = +62$	$Z_5 = +54$
p_6	$X_6 = +26$	$Y_6 = +52$	$Z_6 = +36$
p_7	$X_7 = +26$	$Y_7 = +100$	$Z_7 = +36$

SOLVED PROBLEMS

PROBLEM 17.1. *A cube, 25 mm edge, is placed centrally on the top of another square block, of 40 mm edge and 15 mm thick. Draw the isometric drawing of the two solids.*

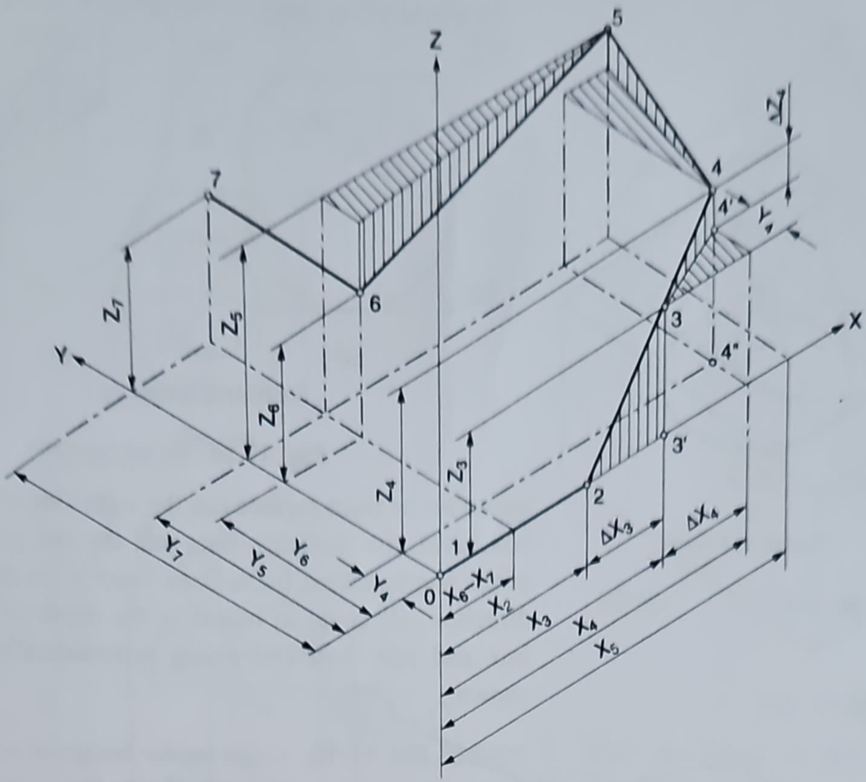


Fig. 17.33.

SOLUTION: Refer *Fig. 17.34.*

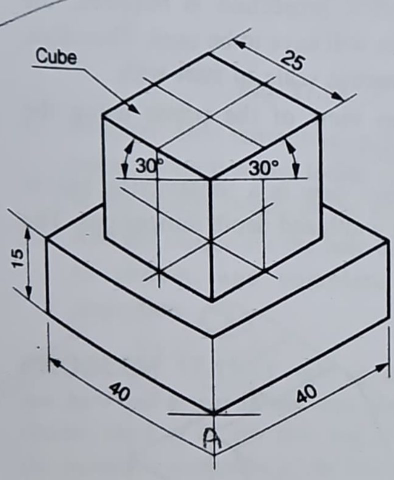


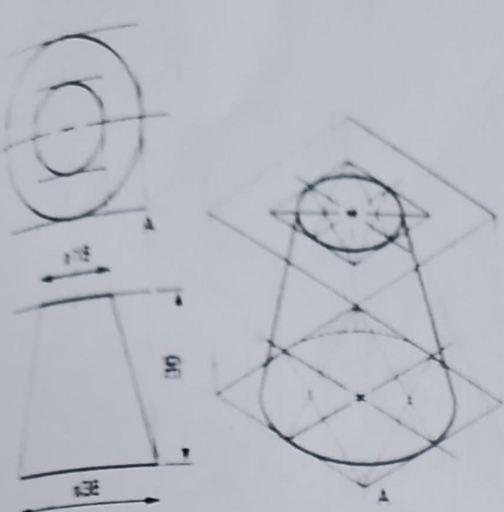
Fig. 17.34.

1. Draw the isometric drawing, using full scale, of the cube following the box-method.
2. Draw two lines at its base surface, parallel to the sides of the surface.
3. Plot points at a distance of half the length of the edge of block, *i.e.*, 20 mm, along these lines on either side of the centre.
4. Through these points draw isometric lines to represent the top of the surface of the block.
5. Draw vertical lines through the corner point of this surface and cut along each of these lines a distance equal to the thickness of the block, *i.e.*, 15 mm.
6. Complete the isometric drawing of the block by drawing isometric lines through these points.
7. Only visible lines or their visible portions need be shown in firm lines.

PROBLEM 17.2. *Draw the isometric drawing of the frustum of a right regular hexagonal pyramid, side of base hexagon is 20 mm, side of top hexagon is 10 mm and height of the frustum is 40 mm.*

SOLUTION: Refer *Fig. 17.35. (a), (b).*

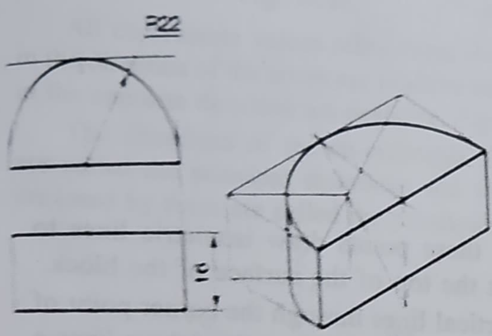
1. Draw the top view of the frustum and enclose the two hexagons, that represent the top and bottom surface of the frustum, in enclosing rectangles.
2. The enclosing rectangle for the base hexagon gives width of the box is same as the height of the frustum.



(a) (b)

Frustum of A Cone

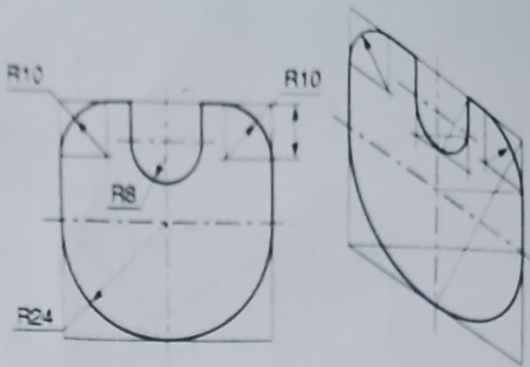
Fig. 17.36. (4)



(a) (b)

Half Cylindrical Disc

Fig. 17.36. (5)



(a) (b)

Sheet Metal Templet

Fig. 17.36. (6)

locate on its front surface (on the right side of the box here), the isometric lines and the end points of the non-isometric lines. Draw lines in the depth direction, all equal in length to the depth of the box and join their end points systematically as shown. Q-2

PROBLEM 17.10. *A right regular hexagonal prism, edge of base 20 mm and height 50 mm, has a circular hole of ⌀ 20 mm, drilled centrally through it, along its axis. Draw its isometric projection.*

SOLUTION: Refer *Fig. 17.38.*

1. As the isometric projection is required, the isometric scale will have to be used. Therefore, draw the isometric scale to start with.
2. Draw the top view of the prism using the isometric scale.
3. Enclose the top view in a construction box to determine its width and depth dimensions. The

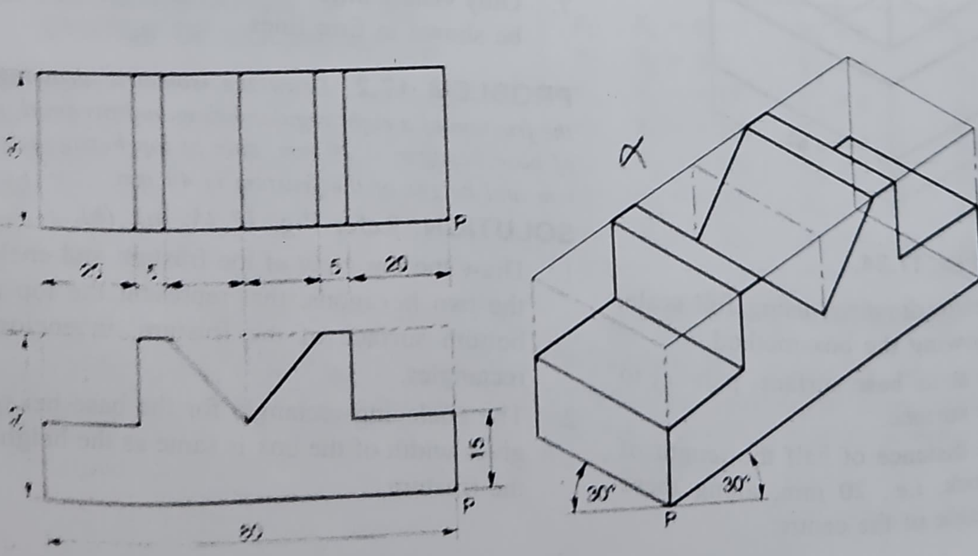


Fig. 17.37.

height of the box will be equal to the height of the prism.

2

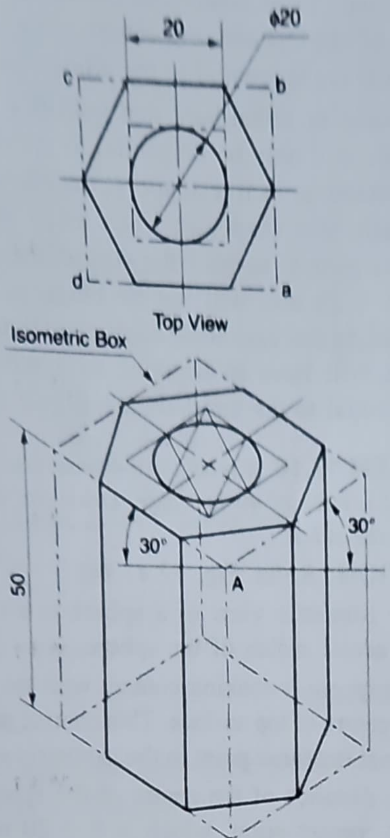


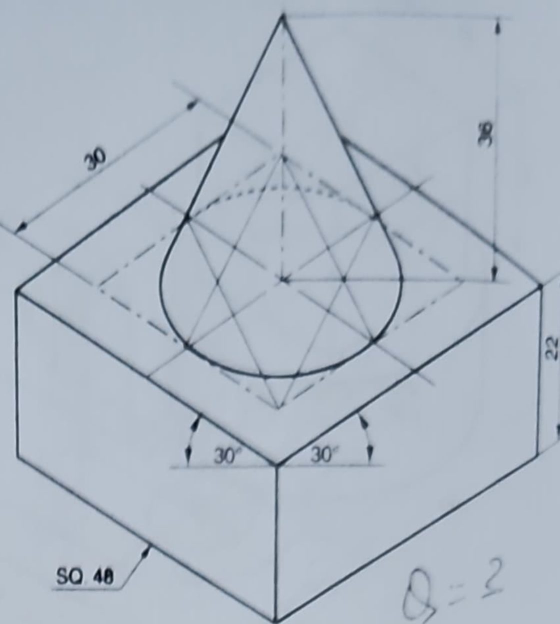
Fig. 17.38.

4. Prepare the isometric projection of the construction box and locate on it the positions of the corner points of the hexagon.
5. Add the ellipse for the hole circle in top view, as shown, and complete the isometric projection. 3

PROBLEM 17.11. *A right circular cone of ϕ 30 mm base and height 36 mm rests centrally on top of a square block of 48 mm side and 22 mm thick. Draw the isometric projection of the two solids.*

SOLUTION: Refer *Fig. 17.39.*

1. Draw the isometric scale to start with.
2. Draw the isometric projection of the block, using the isometric scale.
3. Draw the two centre lines on the top surface of the block.
4. Draw the rhombus symmetrically about the centre point of the top surface of the block, as shown, for the cone base.



Q=2

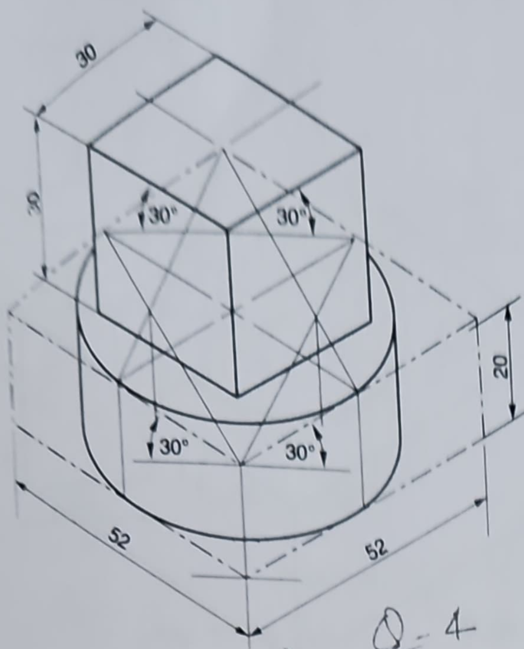
Fig. 17.39.

5. Draw the ellipse for the cone base, using the four centre method.
6. Draw a vertical line through the centre point and cut along it a distance equal to the height of the cone, using isometric scale. Join this point to the ellipse tangentially, on the two sides, as shown.
7. Finish the view by making the visible portions of lines firm. 4

PROBLEM 17.12. *A cube, of 30 mm edge, is placed centrally on top of a cylindrical block of ϕ 52 mm and 20 mm height. Draw the isometric drawing of the solids.*

SOLUTION: Refer *Fig. 17.40.*

1. Draw the isometric box, using full size scale, for the block, since we need to draw the isometric drawing.
2. Draw the two centre lines for its top surface and draw the ellipse for the top of the block, around this centre point, using the four-centre method.
3. Locate the corresponding points, to be used as centres of the four arcs for the base of the block, by offsetting in the vertical downward direction from their positions in the top surface ellipse.
4. Draw the visible part of the base ellipse using the principle of symmetry, as shown.
5. Draw the isometric of the cube coinciding the centre of its base surface with the centre of the top surface of the block, as shown.



Q- 4

Fig. 17.40.

PROBLEM 17.13. *In Fig. 17.41 at (1), on left side, are shown two views of an object. Draw its isometric view.*

SOLUTION: Refer *Fig. 17.41. (1)*. All necessary construction lines are shown to render the solution easy to understand. The interpretation of the solution is left to the students. Note that the isometric scale will have to be used.

PROBLEM 17.14. *Two orthographic views of an object are shown in Fig. 17.41 (2) (a). Draw its isometric view.*

SOLUTION: Refer *Fig. 17.41 (2) (b)*. The interpretation of the solution is left to the reader.

PROBLEM 17.15. *Front view of a cut sphere is shown in Fig. 17.41 (3) (a). Draw its isometric view.*

SOLUTION: Refer *Fig. 17.41 (b)* and *(c)*.

1. Draw the circle for the part bottom view of the cut sphere, below the given front view.
2. Draw true ellipse, using offsets, as shown for isometric view of this circle. See that its enclosing square has a side equal to **b.** The centre of the ellipse is O_1.
3. In front view the height of the centre O of the sphere is 12 mm above the centre O_1 of the cut base.
4. Draw a vertical line through O_1 in the ellipse drawn and cut along it a distance equal to the isometric equivalent of 12 mm as shown by a, to locate the centre O of the sphere in isometric view.
5. Then with O as centre and the radius R = 25 mm of the sphere, on actual scale, draw a circular arc tangential to the ellipse, as shown.
6. It should be noted here that with R = 25 mm the arc will only be tangential to the ellipse if it is drawn to its true shape. If the ellipse is not actually true shape, as when using the four-centre method to draw the ellipse, then the arc of R = 25 mm will not be tangential to the ellipse. In that case some adjustment in the value of R will have to be done to make the arc tangential to the approximate ellipse.

PROBLEM 17.16. *A sphere of ϕ 40 mm rests centrally on top of a cube of 40 mm side. Draw the isometric views of the solids.*

SOLUTION: Refer *Fig. 17.41 (4)*.

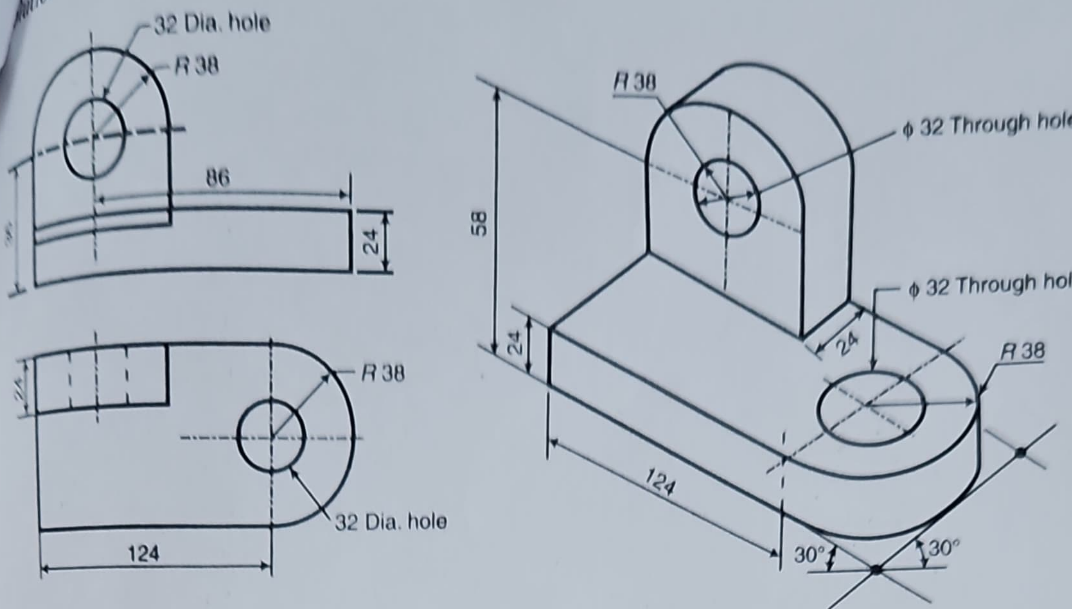
1. The isometric view of a sphere is a circle of the actual radius of the sphere.
2. The sphere is making contact with the cube at the centre of top surface. This contact point will stay at the same point in the isometric view too.
3. The distance of the centre of the sphere from this contact point, which is R = 20 mm, will remain only equal to the isometric equivalent of the radius value, shown as r in isometric view.
4. Draw the isometric view of the cube, using isometric scale.
5. Locate the centre point A of its top surface by drawing two centre lines in isometric directions.
6. Draw a vertically upward line through A and cut along it a distance equal to the isometric equivalent of the radius R = 20 mm of the sphere to locate point O. Then O is the centre of the sphere.
7. Then with O as centre and R = 20 mm, the actual radius of the sphere, draw a circle to represent the sphere.
8. Finish the view by marking the visible part of the cube firm.

PROBLEM 17.17. *A hemisphere of ϕ 36 mm rests on its circular base on the top of a cube of 36 mm side. Draw the isometric projection of the solids.*

Q-5

ple 14.35 Figure 14.45 (a) shows the top view and front view of an object. Draw sometric view of the object.

lution: (Refer Figure 14.45 (b))



(a) Top and front view

(b) Isometric view

Figure 14.45

Q-6

Example 14.36 Draw the isometric projection of a circle having diameter 30 mm and its surface parallel to the H.P.

Solution: **Steps for Construction:**

- Take a horizontal line *XY* and then take a point *A* on it.
- At point *A*, draw line *AB* and *AD* of length 30 mm making an angle of 30° with *XY* line.
- Through *B*, draw a line *BC* parallel to *AD* and through *D*, draw a line *DC* parallel to *AB*.
- These lines intersect each other at *C*. Join *BC* and *DC*. Complete the *ABCD* square.
- Mark the mid points of all sides of the square as *H*, *F*, *G*, and *E*. Join *AC*, *BD*, *EC*, *EF*, *HC*, *HG*, *AG* and *AF*.
- Taking *A* as center and radius *AG* or *AF*, draw arc *GF*.
- Taking *C* as center and radius *CE* or *CH*, draw arc *EH*. Taking *M* as center and radius *MG* or *ME*, draw arc *EG*.

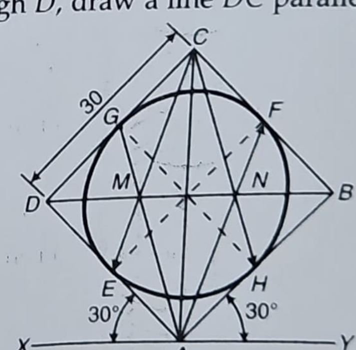


Figure 14.46 Isometric projection of circle.